1. Übung (15. Oktober 2015)

Formale Übersetzungsmodelle

**Task 1 (Ranked alphabets and trees)**

Consider the following trees:

\[
\begin{align*}
\xi_1 &= \sigma \gamma \beta \alpha \alpha \\
\xi_2 &= \sigma \beta \gamma \alpha
\end{align*}
\]

(a) Give \(\text{height}(\xi_i)\), \(\text{size}(\xi_i)\), \(\text{pos}(\xi_i)\), \(\text{sub}(\xi_i)\) for \(i \in \{1, 2\}\).

(b) Define minimal ranked alphabets \(\Delta_1\) and \(\Delta_2\) such that \(\xi_1 \in T_{\Delta_1}\) and \(\xi_2 \in T_{\Delta_2}\).

(c) Extend the intersection, union, and subset relation to ranked alphabets.

(d) Prove or refute: There is a ranked alphabet \(\Gamma\) such that \(\xi_1, \xi_2 \in T_{\Gamma}\).

**Task 2 (Definition by structural induction)**

Let \(\xi \in T_{\Sigma}, w \in \text{pos}(\xi), \zeta \in T_{\Sigma}(X_k), \) and \(\zeta'_1, ..., \zeta'_k \in T_{\Sigma}(A)\). Define the following characteristics of \(\xi\) and \(\zeta\) by structural induction:

(a) \(\xi(w)\), the label of \(\xi\) at position \(w\),

(b) \(\xi|_w\), the subtree of \(\xi\) at position \(w\),

(c) \(\xi[\zeta]|_w\), the tree obtained by substituting the subtree of \(\xi\) at position \(w\) with \(\zeta\),

(d) \(\text{yield}(\xi)\), the sequence of leaves of \(\xi\) from left to right,

(e) \(\zeta[\zeta'_1, ..., \zeta'_k]\), the tree obtained from \(\zeta\) by substituting \(x_i\) by \(\zeta'_i\) for every \(i \in \{1, ..., k\}\).

**Task 3 (Proof by structural induction)**

Let \(\xi, \zeta \in T_{\Sigma}(A)\) and \(w \in \text{pos}(\xi)\). Prove or refute the following statements:

(a) \(\xi(w) = \xi|_w(\varepsilon)\).

(b) \((\xi[\zeta]|_w)|_w = \zeta\).

(c) \(|\text{pos}(\xi)| = |\text{sub}(\xi)|\).

**Note** The tutorial’s time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.