Formale Übersetzungsmodelle

**Task 14 (BOT \circ HOM \subseteq BOT)**

Let \( B = (Q, \Sigma, \Delta, F, R) \) be a bu-tt and \( H = (\{\ast\}, \Delta, \Omega, \{\ast\}, R_H) \) a tree homomorphism. Also let \( X_{\text{max}} = \{x_i \mid i \in \text{max rank}(\Sigma)\} \). Define the bottom-up tree homomorphism \( H' = (\{\ast\}, \Delta, \Omega \cup X_{\text{max}}, \{\ast\}, R_H) \) and the bottom-up tree transducer \( \hat{B} = (Q, \Sigma, \Omega, F, \hat{R}) \) where

\[
\begin{align*}
\sigma(q_1(x_1),\ldots,q_k(x_k)) &\rightarrow q(u') \in R \land u'[\ast(x_1),\ldots,\ast(x_k)] \Rightarrow *_{H'} * (t') \\
\Leftrightarrow \sigma(q_1(x_1),\ldots,q_k(x_k)) &\rightarrow q(t') \in \hat{R}.
\end{align*}
\]

Show that for every \( s \in T_{\Sigma}, q \in Q, \) and \( t \in T_{\Delta} \) the following equivalence holds:

\[
s \Rightarrow_B^* q(t) \iff \exists u \in T_{\Delta} : s \Rightarrow_B^* q(u) \land u \Rightarrow_H^* t.
\]

**Task 15 (Bimorphism characterization of BOT)**

Recall the decomposition result \( \text{BOT} \subseteq \text{REL} \circ \text{FTA} \circ \text{HOM} \). For every bottom-up tree transducer \( B \) we can construct a bottom-up tree relabeling \( B_1 \), a bottom-up finite state tree automaton \( B_2 \), and a bottom-up tree homomorphism \( B_3 \) such that \( \tau(B) = \tau(B_1) \circ \tau(B_2) \circ \tau(B_3) \).

(a) Show that \( (\tau(B_1))^{-1} \in \text{HOM} \).

(b) Give a bimorphism characterization of \( \text{BOT} \), i.e., give a formal definition of (the syntax of) bimorphisms, and its induced tree transformation. Show that the class of tree transformations induced by bimorphisms subsumes \( \text{BOT} \).

(c) Consider the bu-tt \( B = (\{\ast, q, q_f\}, \Sigma, \Sigma, \{q_f\}, R) \) where \( \Sigma = \{\alpha^{(0)}, \beta^{(0)}, \gamma^{(1)}, \sigma^{(2)}\} \) and

\[
R = \{ \alpha \rightarrow q(\alpha), \alpha \rightarrow \ast(\alpha), \beta \rightarrow \ast(\beta), \gamma(q(x_1)) \rightarrow q(\gamma(x_1)), \gamma(\ast(x_1)) \rightarrow \ast(\gamma(x_1)), \sigma(\ast(x_1),q(x_2)) \rightarrow q_f(x_1), \sigma(\ast(x_1),\ast(x_2)) \rightarrow \ast(\sigma(x_1,x_2)) \}
\]

Give a bimorphism \( B = (A, \varphi, \psi) \) such that \( \tau(B) = \tau(B) \).

Give a derivation of \( \xi = \sigma(\gamma(\beta), \gamma(\alpha)) \) in \( B \).

Give a tree \( \zeta \in T_{\Omega} \) such that \( \zeta \in L(A) \) and \( \varphi(\zeta) = \xi \).