Formale Baumsprachen

Task 4 (bu-det fta)
Let \( \Sigma = \{ \sigma(2), \alpha(0), \beta(0) \} \) and \( \Delta = \{ \sigma(2), \gamma(1), \alpha(0) \} \) be ranked alphabets. Give deterministic bu- 
a (a) \( L_1 = \{ \xi \in T_\Sigma \mid \xi \text{ contains at least one } \alpha \text{ and one } \beta \} \), 
(b) \( L_2 = \{ \xi \in T_\Sigma \mid \xi \text{ contains an even number of } \alpha \text{ symbols} \} \), and 
(c) \( L_3 = \{ \sigma(t_1, \sigma(t_2, \ldots \sigma(t_n, \alpha) \ldots)) \in T_\Delta \mid n \in \mathbb{N}, t_1, \ldots, t_n \in T_{\{\gamma(1), \alpha(0)\}} \} \).

Task 5 (string automata I)
Recall the concept of string automata. Let \( \Sigma \) be an alphabet and \( \# \notin \Sigma \). We define the ranked 
alphabet \( \Sigma^\# = \Sigma^{(0)} \cup \Sigma^{(1)} \) where \( \Sigma^{(0)} = \{ \# \} \) and \( \Sigma^{(1)} = \Sigma \). Moreover, we define the \( \Sigma^\# \)- 
algebra \( (\Sigma^*, \theta) \) where \( \theta(\#) = \varepsilon \) and \( \theta(a)(w) = wa \) for every \( a \in \Sigma \) and \( w \in \Sigma^* \).

(a) Show that \( \Sigma^* \) is initial in the class of \( \Sigma^\# \)-algebras.
(b) We consider \( \Sigma = \{ a, b \} \) and the language \( L = \{ a^n b^m \mid n, m \in \mathbb{N} \} \). Sketch the diagram of a 
total deterministic finite-state automaton accepting \( L \) and model the transition table using 
a finite \( \Sigma^\# \)-algebra \( Q \). How can we interpret the uniquely determined homomorphism 
\( h: \Sigma^* \to Q \)?
(c) Convince yourself that any total deterministic finite-state automaton can be modeled as 
a quadruple \( A = (Q, \Sigma, \theta, F) \) where \( (Q, \theta) \) is a finite \( \Sigma^\# \)-algebra and \( F \subseteq Q \). Define the 
language accepted by \( A \) using the homomorphism \( h: \Sigma^* \to Q \).

Task 6 (string automata II)
Let \( \Sigma = \{ a, b \} \) be an alphabet.

(a) Give a finite state automaton \( A = (Q, \Sigma, q_0, F) \) that recognizes 
\( L = \{ w \in \Sigma^* \mid |w|_a - |w|_b \mod 2 \equiv 0 \} \).
(b) Describe \( L \) using a homomorphism between the free monoid \( (\Sigma^*, \circ, \varepsilon) \) and the monoid 
\( (\{0,1\}^Q \times Q, \times, 1_{Q \times Q}) \).
(c) Describe \( L \) using a monoid with carrier \( (\Sigma^*)^{Q \times Q} \).

Note  The tutorial’s time might not suffice for presenting all solutions. Please prepare to ask 
for the solutions you are most interested in.