Task 7 (regular tree grammars)

(a) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be a ranked alphabet. Give regular tree grammars $G_1$ and $G_2$ with

- $L(G_1) = \{\xi \in T_\Sigma \mid \xi \text{ contains exactly one } \sigma\}$
- $L(G_2) = \{\xi \in T_\Sigma \mid \xi \text{ contains the pattern } \sigma(\_, \gamma(\_)) \text{ at least twice}\}$.

(b) Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet and $G = (N, \Sigma, Z, P)$ a regular tree grammar where $N = \{Z, A, B, C\}$ and

$$P = \begin{cases} Z \to \sigma(A, B, C), & Z \to B, & A \to \alpha, & A \to B, \\ B \to \beta, & B \to A, & B \to C, & C \to C. \end{cases}$$

Use the construction from the lecture to give a regular tree grammar in normal form equivalent to $G$.

Task 8 (relatedness)

(a) Give a bu-det fta that is related to the normal form regular tree grammar constructed in Exercise 7 (b).

(b) Give a regular tree grammar that is related to the normal form bu-det fta $M = (Q, \Sigma, \tau, \{q_0\})$ where $Q = \{0, 1\}$, $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$, $q_0 = 0$, and $\tau$ is given by

$$\tau_\alpha() = 1, \quad \tau_\beta() = 0, \quad \text{and} \quad \tau_\sigma(p, q) = (p + q) \% 2 \quad \text{for each } p, q \in Q.$$

Task 9 (tree manipulation)

Let $\Sigma$ be a ranked alphabet and $H$ be a set. Prove or refute the following statements for every $\xi \in T_\Sigma(H)$:

(a) $\forall w \in \text{pos}(\xi): \text{pos}(\xi|_w) \subseteq \text{pos}(\xi)$,

(b) $\forall w \in \text{pos}(\xi): \text{sub}(\xi|_w) \subseteq \text{sub}(\xi)$,

(c) $\forall w \in \text{pos}(\xi), \zeta \in T_\Sigma(H): \text{size}(\xi[\zeta|_w]) = \text{size}(\xi) + \text{size}(\zeta) - \text{size}(\xi|_w)$.

Note The tutorial’s time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.