Task 1 (ranked alphabets and trees)
Consider the following trees:

\[
\begin{align*}
\xi_1 &= \sigma \left/ \gamma \alpha \right/ \beta \alpha \\
\xi_2 &= \sigma \left/ \beta \right/ \gamma \alpha
\end{align*}
\]

(a) Give \(\text{height}(\xi_i), \text{size}(\xi_i), \text{pos}(\xi_i), \text{sub}(\xi_i)\) for \(i \in \{1, 2\}\).

(b) Extend the intersection, union, and subset relation to ranked alphabets.

(c) Define minimal ranked alphabets \(\Delta_1\) and \(\Delta_2\) such that \(\xi_1 \in T_{\Delta_1}\) and \(\xi_2 \in T_{\Delta_2}\).

(d) Prove or refute: There is a ranked alphabet \(\Gamma\) such that \(\xi_1, \xi_2 \in T_\Gamma\).

Task 2 (definition by structural induction)
Let \(\Sigma\) be a ranked alphabet, \(A\) a set, \(\xi \in T_{\Sigma}(A), w \in \text{pos}(\xi), \zeta \in T_{\Sigma}(X_k),\) and \(\zeta_1', \ldots, \zeta_k' \in T_{\Sigma}(A)\).
Define the following characteristics of \(\xi\) and \(\zeta\) by structural induction:

(a) \(\text{height}(\xi)\), number of nodes on the longest path in \(\xi\),

(b) \(\text{size}(\xi)\), number of nodes in \(\xi\),

(c) \(\text{pos}(\xi)\), set of positions in \(\xi\),

(d) \(\text{sub}(\xi)\), set of subtrees of \(\xi\),

(e) \(\xi(w)\), the label of \(\xi\) at position \(w\),

(f) \(\xi|_w\), the subtree of \(\xi\) at position \(w\),

(g) \(\xi[\zeta]|_w\), the tree obtained by substituting the subtree of \(\xi\) at position \(w\) with \(\zeta\),

(h) \(\text{yield}(\xi)\), the sequence of leaves of \(\xi\) from left to right, and

(i) \(\zeta[\zeta_1', \ldots, \zeta_k']\), the tree obtained from \(\zeta\) by substituting \(x_i\) by \(\zeta_i'\) for every \(i \in \{1, \ldots, k\}\).

Note The tutorial’s time might not suffice to present all solutions. Please prepare to ask for the solutions you are most interested in.