Formale Übersetzungsmodelle

Task 3 (proof by structural induction)
Let $A$ be a set, $\Sigma$ be a ranked alphabet, $\xi, \zeta \in T_\Sigma(A)$, and $w \in \text{pos}(\xi)$. Prove or refute the following statements:

(a) $\xi(w) = \xi|_w(\varepsilon)$.
(b) $(\xi[\zeta]_w)_w = \zeta$.
(c) $|\text{pos}(\xi)| = |\text{sub}(\xi)|$.
(d) $\text{height}(\xi) = 1 + \max\{|\rho| \mid \rho \in \text{pos}(\xi)\}$.

Task 4 (generalized sequential machines and bu-tt)
Let $G = (Q, \Sigma, \Delta, q_0, F, R)$ be a gsm. Give bu-tts that simulate the run of $G$

(a) on the nodes of monadic trees from front to root.
(b) on the front of trees from left to right.

Task 5 (relabeling and checking)

(a) Give a bu-tt $M_1$ that, for every tree $\xi \in T_\Sigma$, enhances for every position $w \in \text{pos}(\xi)$ the label at $w$ with the last digit of $w$.

(b) Let $\gamma \in \Sigma$. Give a bu-tt $M_2$ that, for every tree $\xi \in T_\Sigma$, replaces the first occurrence (according to depth-first order) of $\gamma$ in $\xi$ by $\gamma_f$ without changing the rest of $\xi$.

Note: The tutorial’s time might not suffice to present all solutions. Please prepare to ask for the solutions you are most interested in.