Task 15 (Symbolic derivation)

Let $k \in \mathbb{N}$ and $\Sigma = \{+(2), \cdot(2), -(1), \sin(1), \cos(1), X^{(0)}\} \cup \{0^{(0)}, \ldots, k^{(0)}\}$ a ranked alphabet. The trees over $\Sigma$ represent a subset of polynomials with natural number coefficients.

(a) Give a td-tt $T$ that computes the symbolic derivation of a given tree.

(b) Give a derivation for $\xi = +(\sin(X), 5), X)$ in $T$.

(c) Why is there no bu-tt $B'$ such that $\tau(T) = \tau(B')$?

(d) Give a bu-tt $B$ that simplifies a given tree according to the units with respect to $\cdot$ and $+$ and the absorbing nature of 0 with respect to $\cdot$, in particular, $B$ should collapse $\cdot(\xi, 1), \cdot(1, \xi), +(\xi, 0)$, or $+(0, \xi)$ to $\xi$ and $\cdot(\xi, 0)$ or $\cdot(0, \xi)$ to 0 for every $\xi \in T_\Sigma$.

(e) Why is there no td-tt $T'$ such that $\tau(B) = \tau(T')$?