8. Übung (8. Dezember 2016)

Formale Übersetzungsmodelle

Task 16 (In-BOT = ln-TOP)

Let \( \Sigma = \{\sigma(2), \alpha^{(0)}\} \), \( \Delta = \{\sigma(2), \gamma^{(1)}, \alpha^{(0)}\} \) be ranked alphabets, and \( \xi = \sigma(\alpha, \alpha, \alpha) \in T_\Sigma \). Consider the linear non-deleting bu-tt \( B = (\{q_0, q_1\}, \Sigma, \Delta, \{q_0\}, R_B) \) and the linear non-deleting td-tt \( T = (\{q_0, q_1\}, \Sigma, \Delta, \{q_0\}, R_T) \) where

\[
R_B = \{ \alpha \rightarrow q_0(\alpha), \quad \sigma(q_0(x_1), q_0(x_2)) \rightarrow q_1(\sigma(x_1, x_2)) \} \quad \text{and} \quad R_T = \{ \begin{array}{ll}
q_0(\alpha) \rightarrow \alpha, & q_0(\sigma(x_1, x_2)) \rightarrow \sigma(q_1(x_2), q_1(x_1)), \\
q_1(\alpha) \rightarrow \alpha, & q_1(\sigma(x_1, x_2)) \rightarrow \sigma(\gamma(q_0(x_1)), \gamma(q_0(x_2)))
\end{array} \}
\]

(a) Give a linear non-deleting td-tt \( T' \) such that \( \tau(B) = \tau(T') \).
Give derivations of \( B \) and \( T' \) on \( \xi \).

(b) Give a linear non-deleting bu-tt \( B' \) such that \( \tau(T) = \tau(B') \).
Give derivations of \( T \) and \( B' \) on \( \xi \).

Task 17 (generalized sequential machines and top-down tree transducers)

GSM is the class of string transformations \( \tau \subseteq \Sigma^* \times \Delta^* \) that are be induced by some gsm.

(a) Give formal definitions for the syntax and derivation relation of a gsm, and the string transformation induced by a gsm.

(b) Prove by construction that GSM is closed under composition.

Hint: Use a product construction where the right hand side of a rule of the first gsm is processed by the second gsm (pipelining).

Let \( G = (Q, \Sigma, \Delta, q_0, F, R) \) be a gsm.

(c) Give a gsm \( G^R \) such that \( \tau(G^R) = \{(w^R_l, w^R_r) : (w_l, w_r) \in \tau(G)\} \) where \( w^R \) denotes the reverse of \( w \).

(d) Give a td-tt that simulates the run of \( G \) on the nodes of monadic trees from root to front.